Taking the Factorial to the n-th Degree

The factorial of a positive integer n, $n!$ is the product of the first n positive integers. It turns out that there is another (rather surprising) way of finding the number $n!$ for any positive n.

We will first explain method for obtaining $n!$

1. Set up an array of $(n+1)$ rows and $(n+1)$ columns. Let A be the array and $A_{i,j}$ be the entry in the i-th row and j-th column of A
2. In the first column, insert the value i^n in the i-th row (or $A_{i,1} = i^n$)
3. In the j-th column, where $j = 2, 3, \cdots, n+1$, insert the difference between the entries in the $(i+1)$-th row and i-th row of the $(j-1)$-th column in the i-th row.
   (In other words, $A_{i,j} = A_{i+1,j-1} - A_{i,j-1}$, i = 1,2, ..., $n - j + 1$, $j = 2, 3, ..., n+1$)

The number in the first row of the $(n+1)$-th column should be the number $n!$

The following formula is another way of describing the above method for computing $n!$

$$n! = \sum_{i=1}^{n} (-1)^{(n-i)} \binom{n}{i} i^n$$

The formula below is the general form of the above formula, where m is any non-negative integer.

$$n! = \sum_{i=1}^{n} (-1)^{(n-i)} \binom{n}{i} (m+i)^n$$

Note that the above formula are not very useful in practice, as it requires more computations than the naive way of computing $n!$.